

4.5 Notes – Parallel and Perpendicular Lines

Key Concept

Write equations of parallel and perpendicular lines.

GOAL Write equations of parallel and perpendicular lines.

Vocabulary

The **converse of a conditional statement** interchanges the hypothesis and conclusion.

Parallel Lines

If two nonvertical lines have the same slope, then they are **parallel**.

If two nonvertical lines are parallel, then they have the same slope.

Perpendicular Lines

Two lines in a plane are **perpendicular** if they intersect to form a right angle.

If two nonvertical lines have slopes that are negative reciprocals, then the lines are perpendicular.

If two nonvertical lines are perpendicular, then their slopes are negative reciprocals.

Key Concept

Write equations of parallel and perpendicular lines.

Common Student Errors

- Students may think slopes of perpendicular lines are negative for both lines

Tip You may want to use “opposite reciprocals” instead of “negative reciprocals” to describe the slopes of perpendicular lines.

- Students may incorrectly give the coefficient of x as the slope of a line written in standard form

Tip Reinforce that an equation must be written in slope-intercept form in order to give the coefficient of x as the slope m .

Example: What is the slope of the line perpendicular to $y = -4x + 3$?

Student response: $m = -\frac{1}{4}$ ✗

Example: What is the slope of the line parallel to $3x - 2y = 1$?

Student response: $m = 3$ ✗

EXAMPLE 1**Write an equation of a parallel line**

Write an equation of the line that passes through (2, 6) and is parallel to the line $y = -x + 2$.

Solution

STEP 1 Identify the slope. The graph of the given equation has a slope of -1 . So, the parallel line through (2, 6) has a slope of -1 .

STEP 2 Find the y -intercept. Use the slope and the given point.

$$y = mx + b \quad \text{Write slope-intercept form.}$$

$$6 = -1(2) + b \quad \text{Substitute } -1 \text{ for } m, 2 \text{ for } x, \text{ and } 6 \text{ for } y.$$

$$8 = b \quad \text{Solve for } b.$$

STEP 3 Write the equation. Use $y = mx + b$.

$$y = -x + 8 \quad \text{Substitute } -1 \text{ for } m \text{ and } 8 \text{ for } b.$$

Exercises for Example 1

Write an equation of the line that passes through the given point and is parallel to the given line.

1. $(9, 2), y = \frac{2}{3}x + 1$

2. $(-3, -4), y = -2x - 1$

EXAMPLE 2**Determine whether lines are parallel or perpendicular**

Determine which of the following lines, if any, are parallel or perpendicular: Line a : $4y - 6x = -8$, Line b : $y = -\frac{2}{3}x + 1$, Line c : $2x + 3y = -15$.

Solution

Find the slopes of the lines.

Line b : The equation is in slope-intercept form. The slope is $-\frac{2}{3}$.

Write the equations for lines a and c in slope-intercept form.

$$\text{Line } a: 4y - 6x = -8$$

$$\text{Line } c: 2x + 3y = -15$$

$$4y = 6x - 8$$

$$3y = -2x - 15$$

$$y = \frac{3}{2}x - 2$$

$$y = -\frac{2}{3}x - 5$$

Lines b and c have a slope of $-\frac{2}{3}$, so they are parallel. Line a has a slope of $\frac{3}{2}$, the negative reciprocal of $-\frac{2}{3}$, so it is perpendicular to lines b and c .

EXAMPLE 3 **Write an equation of a perpendicular line**

Write an equation of the line that passes through $(-2, 1)$ and is perpendicular to the line $y = -\frac{1}{3}x + 2$.

Solution

STEP 1 Identify the slope. The graph of the given equation has a slope of $-\frac{1}{3}$.

Because the slopes of perpendicular lines are negative reciprocals, the slope of the perpendicular line through $(-2, 1)$ is 3.

STEP 2 Find the y -intercept. Use the slope and the given point in $y = mx + b$.

$$1 = 3(-2) + b \quad \text{Substitute 3 for } m, -2 \text{ for } x, \text{ and 1 for } y.$$

$$7 = b \quad \text{Solve for } b.$$

STEP 3 Write the equation. Use $y = mx + b$.

$$y = 3x + 7 \quad \text{Substitute 3 for } m \text{ and 7 for } b.$$

Exercises for Examples 2 and 3

- Determine which of the following lines, if any, are parallel or perpendicular.
Line a : $-3x - 12y = 36$ Line b : $x + 4y = 2$ Line c : $y = 4x$
- Write an equation of the line that passes through $(5, 3)$ and is perpendicular to the line $y = -5x + 3$.