

5.2 Multiplication/Division of inequalities

GOAL Solve inequalities using multiplication and division.

Multiplication Property of Inequality

Multiplying each side of an inequality by a *positive* number produces an equivalent inequality.

Multiplying each side of an inequality by a *negative* number and *reversing the direction of the inequality symbol* produces an equivalent inequality.

Common Student Errors

- Reversing the inequality whenever there is multiplication or division in the problem or whenever there is a negative sign involved in the problem

Tip Remind your students that they should only reverse the inequality symbol if they have to multiply or divide by a negative number. To emphasize these properties, show several examples.

Example: Solve $2x < 4$.

Student response: $2x < 4$

$$x > 2 \quad \times$$

Example: Solve $\frac{x}{2} \geq -3$.

Student response: $\frac{x}{2} \geq -3$

$$x \leq -6 \quad \times$$

EXAMPLE 1**Solve an inequality using multiplication**

Solve the inequality. Graph your solution.

a. $\frac{x}{7} > 3$

b. $\frac{x}{-2} \leq 5$

Solution

a. $\frac{x}{7} > 3$

Write original inequality.

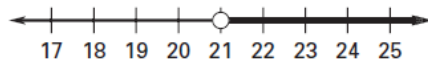
$7 \cdot \frac{x}{7} > 7 \cdot 3$

Multiply each side by 7.

$x > 21$

Simplify.

The solutions are all real numbers greater than 21. Check by substituting a number greater than 21 in the original inequality.



b. $\frac{x}{-2} \leq 5$

Write original inequality.

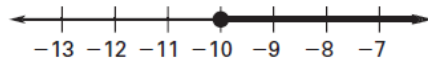
$-2 \cdot \frac{x}{-2} \geq -2 \cdot 5$

Multiply each side by -2 . Reverse inequality symbol.

$x \geq -10$

Simplify.

The solutions are all real numbers greater than or equal to -10 . Check by substituting a number greater than or equal to -10 in the original inequality.

**Exercises for Example 1**

Solve the inequality. Graph your solution.

1. $\frac{m}{4} < -3$

2. $\frac{n}{-6} \leq 4$

3. $\frac{p}{-1.2} > -8$

EXAMPLE 2 **Solve an inequality using division**

Solve $6x > -36$.

Solution

$6x > -36$ Write original inequality.

$\frac{6x}{6} > \frac{-36}{6}$ Divide each side by 6.

$x > -6$ Simplify.

Exercises for Example 2

Solve the inequality.

4. $-3x \leq 9$

5. $18 \geq 9x$

6. $6x < 12$

EXAMPLE 3 **Solve a real-world problem**

A library has \$180 to buy new books. The books cost \$9 each. Write and solve an inequality to find the possible number of books that can be bought for the library.

Solution

The total cost of the books can be at most the amount of money available. Write a verbal model for the situation. Then write an inequality.

$$\text{Books} \cdot \text{Cost per book} \leq 180$$

$$b \cdot 9 \leq 180$$

$$b \leq 20$$

The library can afford to buy at most 20 books.

Exercises for Example 3

- In Example 3, suppose the library has \$120 to spend and that books cost \$8 each. Write and solve an inequality to find the possible number of books the library can buy.
- Three sisters want to buy a PDA for their father for Father's Day. The least expensive PDA in the store is \$360. Write and solve an inequality to find the least amount of money each girl would have to contribute, if each contributes an equal amount.