5.4 Compound Inequalities

GOAL **Solve and graph compound inequalities.**

Vocabulary

A **compound inequality** consists of two separate inequalities joined by *and* or *or*.

Common Student Errors

- Not reversing both inequality symbols

  Tip Remind students to reverse both inequality symbols when multiplying or dividing a compound inequality by a negative number.

  Example:

  $2 < -2x < 6$

  $-1 < x > -3$

- Confusing graphs of inequalities involving *or* with inequalities involving *and*

  Tip Stress to students that compound inequalities involving *and* have solutions that intersect and most compound inequalities involving *or* have gaps with arrows pointing in opposite directions.

  Example:

  $x < 1$ *or* $x > 3$

EXAMPLE 1 Write and graph a compound inequality

Translate the verbal phrases into an inequality. Then graph the inequality.

a. All real numbers that are less than or equal to 7 *or* greater than or equal to 10.

Inequality: $x \leq 7$ *or* $x \geq 10$

b. All real numbers that are greater than -1 *and* less than or equal to 1.

Inequality: $-1 < x < 1$

Exercises for Example 1

Translate the verbal phrases into an inequality. Then graph the inequality.

1. All real numbers that are less than -3 *or* greater than 0.
2. All real numbers that are less than 9 *and* greater than or equal to 7.
3. All real numbers that are greater than or equal to 14 *or* less than or equal to 10.

EXAMPLE 2 Solve a compound inequality with *and*

Solve $7 \leq x - 4 \leq 12$. Graph your solution.

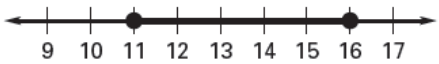
Solution

$$7 \leq x - 4 \leq 12 \qquad \text{Write original inequality.}$$

$$7 + 4 \leq x - 4 + 4 \leq 12 + 4 \qquad \text{Add 4 to each expression.}$$

$$11 \leq x \leq 16 \qquad \text{Simplify.}$$

The solutions are all real numbers greater than or equal to 11 *and* less than or equal to 16.



EXAMPLE 3 Solve a compound inequality with *or*

Solve $3x + 4 < 16$ *or* $5x - 12 > 13$. Graph your solution.

Solution

Solve the two inequalities separately.

$3x + 4 < 16$	*or*	$5x - 12 > 13$	Write original inequality.
$3x + 4 - 4 < 16 - 4$	*or*	$5x - 12 + 12 > 13 + 12$	Use addition or subtraction property of inequality.
$3x < 12$	*or*	$5x > 25$	Simplify.
$\frac{3x}{3} < \frac{12}{3}$	*or*	$\frac{5x}{5} > \frac{25}{5}$	Use division property of inequality.
$x < 4$	*or*	$x > 5$	Simplify.

The solutions are all real numbers less than 4 *or* greater than 5.

Exercises for Examples 2 and 3

Solve the inequality. Graph your solution.

4. $9 < 2x + 3 < 15$

5. $30 \geq -7x - 12 > 16$

6. $28 \leq 4(2x - 3) \leq 68$

7. $3x - 7 < 11$ *or* $x + 4 > 15$

8. $\frac{1}{2}(x + 18) > 6$ *or* $7x + 5 < -51$

9. $3x + 8 > 7x - 12$ *or* $9(x - 2) > 8x - 9$