

5.5 Absolute Value Equations

GOAL Solve absolute value equations.

Vocabulary

An **absolute value equation**, such as $|x| = 3$, is an equation that contains an absolute value expression.

The **absolute deviation** of a number x from a given value is the absolute value of the difference of x and the given value:

$$\text{absolute deviation} = |x - \text{given value}|.$$

Common Student Errors

- Attempting to solve a false equation

Tip Remind students that some absolute value equations have no solution and that the absolute value of an expression can never be negative.

- Thinking that solutions for any absolute value equation are the same number with opposite signs

Tip Remind students that they need to solve two separate equations $ax + b = c$ and $ax + b = -c$ to find the solutions of absolute value equations.

Example: Solve $|x + 4| = -10$.

Student solution:

$$x + 4 = -10 \text{ or } x + 4 = 10$$

$$x = -14 \text{ or } x = 6 \text{ X}$$

Example: Solve $|x + 5| = 7$

Student solution: $x + 5 = 7$

$$x = \pm 2 \text{ X}$$

EXAMPLE 1**Solve an absolute value equation**

Solve the equation.

a. $|x| = 3$

b. $|x + 2| = 9$

Solution**a.** The distance between x and 0 is 3. So, $x = 3$ or $x = -3$. The solutions are 3 and -3 .**b.** Rewrite the absolute value equation as two equations. Then solve each equation separately.

$$|x + 2| = 9$$

Write original equation.

$$x + 2 = 9 \quad \text{or} \quad x + 2 = -9$$

Rewrite as two equations.

$$x = 7 \quad \text{or} \quad x = -11$$

Subtract 2 from each side

The solutions are 7 and -11 . Check your solutions.

CHECK $|x + 2| = 9$

Write original inequality.

$$|7 + 2| \stackrel{?}{=} 9 \qquad |-11 + 2| \stackrel{?}{=} 9$$

Substitute for x .

$$|9| \stackrel{?}{=} 9 \qquad |-9| \stackrel{?}{=} 9$$

Add.

$$9 = 9 \checkmark$$

$$9 = 9 \checkmark$$

Simplify. The solution checks.

Exercises for Example 1

Solve the equation.

1. $|x| = 0.4$

2. $|x - 4| = 13$

3. $|2x - 1| = 7$

EXAMPLE 2**Rewrite an absolute value equation**

Solve $\frac{1}{2}|3x - 6| + 7 = 13$.

SolutionFirst, rewrite the equation in the form $|ax + b| = c$.

$$\frac{1}{2}|3x - 6| + 7 = 13$$
 Write original equation.

$$\frac{1}{2}|3x - 6| = 6$$
 Subtract 7 from each side.

$$|3x - 6| = 12$$
 Multiply each side by two.

Next, solve the absolute value equation.

$$|3x - 6| = 12$$
 Write absolute value equation.

$$3x - 6 = 12 \quad \text{or} \quad 3x - 6 = -12$$
 Rewrite as two equations.

$$3x = 18 \quad \text{or} \quad 3x = -6$$
 Add 6 to each side.

$$x = 6 \quad \text{or} \quad x = -2$$
 Divide each side by 3.

The solutions are 6 and -2 .**EXAMPLE 3****Decide if an equation has no solution**

Solve $|2x - 1| + 4 = 3$, if possible.

Solution

$$|2x - 1| + 4 = 3$$
 Write original equation.

$$|2x - 1| = -1$$
 Subtract 4 from each side.

The absolute value of a number is never negative. So, there are no solutions.

Exercises for Examples 2 and 3

Solve the equation, if possible.

4. $2|x - 1| - 5 = 9$

5. $5|x - 4| + 11 = 8$

6. $\frac{1}{5}|2x - 3| - 4 = 1$