

5.6 Notes - Compound Absolute Values

GOAL Solve absolute value inequalities.

Common Student Errors

- Incorrectly solving inequalities of the form $|ax + b| > c$

Tip Point out that this is incorrect because the “inequality” does not make sense—i.e., -7 is not greater than 7 .

Example: Solve $|x + 3| > 7$.

Student solution: $-7 > x + 3 > 7$
 $-10 > x > 4$ ✗

EXAMPLE 1 Solve an absolute value inequality

Solve the inequality. Graph your solution.

a. $|x| \leq 5$

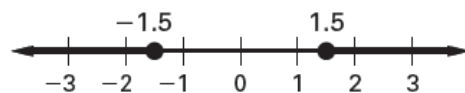
b. $|x| \geq 1.5$

Solution

- a. The distance between x and 0 is less than or equal to 5 . So, $-5 \leq x \leq 5$. The solutions are all real numbers greater than or equal to -5 and less than or equal to 5 .



- b. The distance between x and 0 is greater than or equal to 1.5 . So, $x \geq 1.5$ or $x \leq -1.5$. The solutions are all real numbers greater than or equal to 1.5 or less than or equal to -1.5 .



Exercises for Example 1

Solve the inequality. Graph your solution.

1. $|x| > 2$

2. $|x| \leq 0.2$

3. $|x| \geq \frac{1}{4}$

EXAMPLE 2**Solve an absolute value inequality**

Solve $|-x + 2| < 7$. **Graph your solution.**

Solution

$$|-x + 2| < 7$$

Write original equation.

$$-7 < -x + 2 < 7$$

Rewrite as a compound inequality.

$$-9 < -x < 5$$

Subtract 2 from each expression.

$$9 > x > -5$$

Divide each side by -1 . Reverse inequality symbol.

The solutions are all real numbers greater than -5 *and* less than 9 .



EXAMPLE 3**Solve an absolute value inequality**

Solve $|5x - 1| - 4 \geq 7$. **Graph your solution.**

Solution

$$|5x - 1| - 4 \geq 7$$

Write original equation.

$$|5x - 1| \geq 11$$

Add 4 to each side.

$$5x - 1 \geq 11 \quad \text{or} \quad 5x - 1 \leq -11$$

Rewrite as a compound inequality.

$$5x \geq 12 \quad \text{or} \quad 5x \leq -10$$

Subtract 2 from each expression.

$$x \geq 2.4 \quad \text{or} \quad x \leq -2$$

Divide each expression by 5.

The solutions are all real numbers greater than or equal to 2.4 *or* less than or equal to -2 . Check several solutions in the original inequality.

**Exercises for Examples 2 and 3**

Solve the inequality. Graph your solution.

4. $|x - 4| < 6$

5. $|8x + 5| > 17$

6. $|x - 11| + 8 > 13$

7. $2|x - 1| < 10$

8. $4|x + 7| - 3 \leq 5$

9. $\frac{1}{3}|x - 3| + 2 \leq 4$

Exercises for Example 4

Simplify the expression.

1. $13x - 8 + x^2 + 6x$

2. $9w^2 + 4w - 8w + 3w^2$

2. $12x + 5x - 3 + 4$

4. $10y \times 4y - 8y + 3y$