

GOAL Graph linear inequalities in two variables.

Vocabulary

A **linear inequality in two variables**, such as $x - 3y < 6$, is the result of replacing the $=$ sign in a linear equation with $<$, \leq , $>$, or \geq .

In a coordinate plane, the **graph of an inequality in two variables** is the set of points that represent all solutions of the inequality.

Common Student Errors

- Always using $(0, 0)$ as a test point

Tip Remind students that the test point cannot be on the boundary line of the half-plane. Use an example where the boundary line passes through the origin to show students how to pick a test point.

Inequalities of the form $y > ax$, $y < ax$, $y \geq ax$, and $y \leq ax$, where a is a real number, have boundary lines that pass through the origin.

EXAMPLE 1 Checking solutions

Tell whether the ordered pair is a solution of the inequality.

a. $3x - y > 7$; $(4, 3)$

b. $\frac{1}{2}x - 3y \leq 8$; $(10, -3)$

Solution

- a. Check whether the ordered pair is a solution of the inequality.

$$\begin{array}{ll} 3x - y > 7 & \text{Write original inequality.} \\ 3(4) - 3 \stackrel{?}{>} 7 & \text{Substitute 4 for } x \text{ and 3 for } y. \\ 9 > 7 \checkmark & \text{Simplify.} \end{array}$$

So, $(4, 3)$ is a solution of $3x - y > 7$.

- b. $\frac{1}{2}x - 3y \leq 8$ Write original equation.

$$\begin{array}{ll} \frac{1}{2}(10) - 3(-3) \stackrel{?}{\leq} 8 & \text{Substitute 10 for } x \text{ and } -3 \text{ for } y. \\ 14 \leq 8 & \text{Simplify.} \end{array}$$

Because $14 \leq 8$ is not true, $(10, -3)$ is *not* a solution of the inequality.

Exercises for Example 1

Tell whether the ordered pair is a solution of $-5x + 2y < 11$.

1. $(2, 6)$

2. $(-1, 4)$

3. $(-3, -4)$

Tell whether the ordered pair is a solution of $\frac{1}{3}x + 4y \geq 16$.

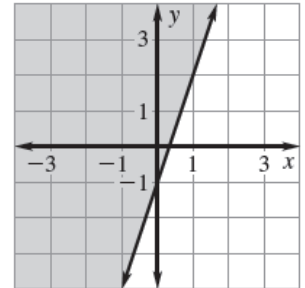
4. $(12, 4)$

5. $(3, 3)$

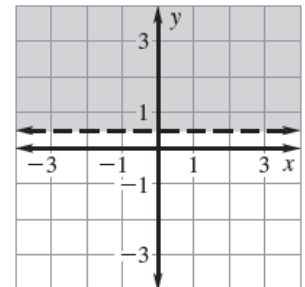
6. $(18, 2)$

EXAMPLE 2 Graph a linear inequality in two variables**Graph the inequality $3x - y \leq 1$.****Solution****STEP 1** Graph the equation $3x - y = 1$.
The inequality is \leq , so use a solid line.**STEP 2** Test $(0, 0)$ in $3x - y \leq 1$.

$$\begin{aligned}3(0) &\stackrel{?}{\leq} 1 \\0 &\leq 1 \checkmark\end{aligned}$$

STEP 3 Shade the half-plane that contains $(0, 0)$, because $(0, 0)$ is a solution of the inequality.**EXAMPLE 3** Graph a linear inequality in one variable**Graph the inequality $y > \frac{1}{2}$.****Solution****STEP 1** Graph the equation $y = \frac{1}{2}$. The inequality is $>$ so use a dashed line.**STEP 2** Test $(0, 0)$ in $y > \frac{1}{2}$. You substitute only the y -coordinate because the inequality does not have the variable x .

$$0 \stackrel{?}{>} \frac{1}{2}$$

STEP 3 Shade the half-plane that does *not* contain $(0, 0)$, because $(0, 0)$ is not a solution of the inequality.**Exercises for Examples 2 and 3****Graph the inequality.**

7. $x + y \geq -2$

8. $5x - 2y < 6$

9. $x \leq 1$