6.3 Linear Combination AKA Addition

GOAL **Solve linear systems by elimination.**

Common Student Errors

- Using the elimination method before arranging like terms

  Tip Stress to students that when using the elimination method, the like terms and equal signs must be in the same columns. Give several examples in which students must arrange like terms of a system before solving.

Example: Solve $x + 2y = 5$
$-x = y + 1$.

Student solution:

Add equations: $2y + y = 6$

Solve for y: $y = 2$

Substitute and solve for x: $x + 2(2) = 5$
$x = 1$

Solution: $(1, 2)$

EXAMPLE 1 Use addition to eliminate a variable

Solve the linear system: $2x + 4y = 2$ Equation 1

$4x - 4y = 16$ Equation 2

Solution

STEP 1 **Add** the equations to eliminate one variable.

STEP 2 **Solve** for x.

$$\begin{array}{r} 2x + 4y = 2 \\ \underline{4x - 4y = 16} \\ 6x \quad\quad = 18 \\ x = 3 \end{array}$$

STEP 3 **Substitute** 3 for x in either equation and solve for y.

$2x + 4y = 2$	Write Equation 1.
$2(3) + 4y = 2$	Substitute 3 for x.
$y = -1$	Solve for y.

The solution is $(3, -1)$.

CHECK Substitute 3 for x and -1 for y in each equation.

Equation 1	**Equation 2**
$2x + 4y = 2$	$4x - 4y = 16$
$2(3) + 4(-1) \stackrel{?}{=} 2$	$4(3) - 4(-1) \stackrel{?}{=} 16$
$2 = 2$ ✓	$16 = 16$ ✓

EXAMPLE 2 Use subtraction to eliminate a variable

Solve the linear system: $7x + 5y = 18$ Equation 1

$7x - 3y = 34$ Equation 2

Solution

STEP 1 **Subtract** the equations to eliminate one variable.

STEP 2 **Solve** for y.

$$\begin{array}{r} 7x + 5y = 18 \\ \underline{7x - 3y = 34} \\ 8y = -16 \\ y = -2 \end{array}$$

STEP 3 **Substitute** -2 for y in either equation and solve for x.

$7x + 5y = 18$	Write Equation 1.
$7x + 5(-2) = 18$	Substitute -2 for y.
$x = 4$	Solve for x.

The solution is $(4, -2)$.

EXAMPLE 3 Arrange like terms

Solve the linear system: $6x - 4y = 10$ Equation 1

$13y = 6x + 8$ Equation 2

Solution

STEP 1 **Rewrite** Equation 1 so that the like terms are arranged in columns.

$$\begin{array}{l} 6x - 4y = 10 \\ 13y = 6x + 8 \end{array} \quad \Rightarrow \quad \begin{array}{r} 6x - \ 4y = 10 \\ -6x + 13y = 8 \end{array}$$

STEP 2 **Add** the equations. $9y = 18$

STEP 3 **Solve** for y. $y = 2$

STEP 4 **Substitute** 2 for y in either equation and solve for x.

$6x + 4y = 10$	Write Equation 1.
$6x - 4(2) = 10$	Substitute 2 for y.
$x = 3$	Solve for x.

The solution is (3, 2).

Exercises for Examples 1, 2, and 3

Solve the linear system.

1. $5x + 8y = 36$
$7x - 8y = 12$

2. $4x + 5y = 8$
$-4x - 3y = 0$

3. $9x - 8y = 7$
$9x + 2y = -13$

4. $-4x + 7y = 11$
$2x + 7y = 47$

5. $9x + 8y = -30$
$9x = 4y + 42$

6. $5y = 4x + 3$
$7x = 36 - 5y$