Chp 6.5 Notes

GOAL **Identify the number of solutions of a linear system.**

Vocabulary

A linear system with no solution is called an **inconsistent system.**

A linear system with infinitely many solutions is called a **dependent system.**

Common Student Errors

- Thinking lines with the same slope means the system has infinitely many solutions

  Tip Stress that in order for a system to have infinitely many solutions, both the slopes and y-intercepts must be the same.

- Confusing the meanings of false and true statements

  Tip Remind students that a true statement means infinitely many solutions and a false statement means no solution.

Example: How many solutions does the system have? Explain.

$3x - y = -1$
$3x - y = 6$

Student response: Infinitely many solutions because the two lines have the same slope.

This word association may help:

Associate *false*, with *no solution*.
Associate *true for all values*, with *infinitely many solutions*.

EXAMPLE 1 A linear system with no solution

Show that the linear system has no solution.

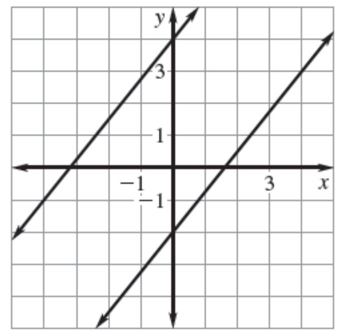
$-5x + 4y = 16$ Equation 1

$5x - 4y = 8$ Equation 2

Solution

Method 1 Graphing

Graph the linear system.



The lines are parallel because they have the same slope but different y-intercepts. Parallel lines do not intersect, so the system has no solution.

Method 2 Elimination

Add the equations.

$$\begin{array}{r} -5x + 4y = 16 \\ 5x - 4y = 8 \\ \hline 0 = 24 \end{array}$$

← This is a false statement.

The variables are eliminated and you are left with a false statement regardless of the values of x and y. This tells you that the system has no solution.

EXAMPLE 2 A linear system with infinitely many solutions

Show that the linear system has infinitely many solutions.

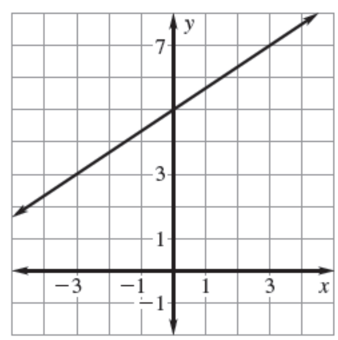
$y = \frac{2}{3}x + 5$ Equation 1

$-2x + 3y = 15$ Equation 2

Solution

Method 1 Graphing

Graph the linear system.



The equations represent the same line, so any point on the line is a solution. So, the linear system has infinitely many solutions.

Method 2 Substitution

Substitute $\frac{2}{3}x + 5$ for y in Equation 2 and solve for x.

$-2x + 3y = 15$	Write Equation 2.
$-2x + 3\left(\frac{2}{3}x + 5\right) = 15$	Substitute $\frac{2}{3}x + 5$ for y.
$-2x + 2x + 15 = 15$	Distributive property
$15 = 15$	Simplify.

The variables are eliminated and you are left with a statement that is true regardless of the values of x and y. This tells you the system has infinitely many solutions.

Exercises for Examples 1 and 2

Tell whether the linear system has *no solution* or *infinitely many solutions.*

1. $-15x + 3y = 6$
$y = 5x + 2$

2. $-4x + y = 5$
$y = 4x + 3$

EXAMPLE 3 Identify the number of solutions

Without solving the linear system, tell whether the linear system has *one solution, no solution,* or *infinitely many solutions.*

a. $7x - 2y = 9$ Equation 1
$7x - 2y = -1$ Equation 2

b. $3x + y = -10$ Equation 1
$-6x - 2y = 20$ Equation 2

Solution

a. $y = \frac{7}{2}x - \frac{9}{2}$ Write Equation 1 in slope-intercept form.

$y = \frac{7}{2}x + \frac{1}{2}$ Write Equation 2 in slope-intercept form.

Because the lines have the same slope but different y-intercepts, the system has no solution.

b. $y = -3x - 10$ Write Equation 1 in slope-intercept form.

$y = -3x - 10$ Write Equation 2 in slope-intercept form.

The lines have the same slope and y-intercept, so the system has infinitely many solutions.

Exercises for Example 3

Without solving the linear system, tell whether the linear system has *one solution, no solution,* or *infinitely many solutions.*

3. $x - 3y = 7$
$4x = 12y + 28$

4. $2x + 3y = 17$
$3x + 2y = 14$

5. $-4x + y = 5$
$-8x - 14y = -28$