

8.1 Notes Polynomials – Adding and Subtracting

GOAL Add and subtract polynomials.

Vocabulary

A **monomial** is a number, a variable, or the product of a number and one or more variables with whole number exponents.

The **degree of a monomial** is the sum of the exponents of the variables in the monomial.

A **polynomial** is a monomial or a sum of monomials, each called a *term* of the polynomial.

The **degree of a polynomial** is the greatest degree of its terms.

When a polynomial is written so that the exponents of a variable decrease from left to right, the coefficient of the first term is called the **leading coefficient**.

A polynomial with two terms is called a **binomial**.

A polynomial with three terms is called a **trinomial**.

Common Student Errors

- Not distributing the negative sign to every term

Tip Remind students to multiply each term of the polynomial by -1 when writing subtraction as addition.

Example: Find the difference.

$$(2x^2 + 3) - (4x^2 + 3x - 1)$$

Student solution:

$$(2x^2 + 3) - (4x^2 + 3x - 1)$$

$$= 2x^2 + 3 - 4x^2 + 3x - 1$$

wrong sign \nearrow wrong sign

$$= -2x^2 + 3x + 2$$

EXAMPLE 1 Rewrite a polynomial

Write $12x^3 - 15x + 13x^5$ so that the exponents decrease from left to right. Identify the degree and the leading coefficient of the polynomial.

Solution

Consider the degree of each of the polynomial's terms.

Degree is 3. Degree is 1. Degree is 5.

$$12x^3 - 15x + 13x^5$$

The polynomial can be rewritten as $13x^5 + 12x^3 - 15x$. The greatest degree is 5, so the degree of the polynomial is 5, and the leading coefficient is 13.

Exercises for Example 1

Write the polynomial so that the exponents decrease from left to right. Identify the degree and the leading coefficient of the polynomial.

1. $9 - 2x^2$

2. $16 + 3y^3 + 2y$

3. $6z^3 + 7z^2 - 3z^5$

EXAMPLE 2 Add polynomials

Find the sum.

a. $(3x^4 - 2x^3 + 5x^2) + (7x^2 + 9x^3 - 2x)$

b. $(7x^2 - 3x + 6) + (9x^2 + 6x - 11)$

Solution

a. Vertical format: Align like terms in vertical columns.

$$\begin{array}{r} 3x^4 - 2x^3 + 5x^2 \\ + \quad 9x^3 + 7x^2 - 2x \\ \hline 3x^4 + 7x^3 + 12x^2 - 2x \end{array}$$

EXAMPLE 3 Subtract polynomials

Find the difference.

a. $(3x^2 - 9x) - (2x^2 - 5x + 6)$

b. $(11x^2 + 6x - 1) - (2x^2 - 7x + 5)$

Solution

a. Vertical format: Align like terms in vertical columns.

$$\begin{array}{r} 3x^2 - 9x \\ - (2x^2 - 5x + 6) \\ \hline \end{array} \longrightarrow \begin{array}{r} 3x^2 - 9x \\ - 2x^2 + 5x - 6 \\ \hline x^2 - 4x - 6 \end{array}$$

Exercises for Examples 2 and 3

Find the sum or difference.

4. $(2a^2 + 7) + (7a^2 + 4a - 3)$

5. $(9b^2 - b + 8) + (4b^2 - b - 3)$

6. $(7c^3 - 6c + 4) - (9c^3 - 5c^2 - c)$

7. $(d^2 - 15d + 10) - (-12d^2 + 8d - 1)$