

8.5 Factoring Trinomials

GOAL Factor trinomials of the form $x^2 + bx + c$.

EXAMPLE 1 Factor when b and c are positive

Factor $x^2 + 10x + 24$.

Solution

Find two positive factors of 24 whose sum is 10. Make an organized list.

Factors of 24	Sum of factors	
24, 1	$24 + 1 = 25$	X
12, 2	$12 + 2 = 14$	X
8, 3	$8 + 3 = 11$	X
6, 4	$6 + 4 = 10$	← correct sum

The factors 6 and 4 have a sum of 10, so they are the correct values of p and q .

$$x^2 + 10x + 24 = (x + 6)(x + 4)$$

CHECK $(x + 6)(x + 4) = x^2 + 4x + 6x + 24$ Multiply binomials.
 $= x^2 + 10x + 24$ ✓ Simplify.

Start with 2 parentheses. () ()

What times what is going to give you the first term?

To get the middle and last term

- What times what is going to give you the last term, but when the same 2 numbers are going to get added, will get you the middle term.

EXAMPLE 3 Factor when b is positive and c is negativeFactor $k^2 + 6k - 7$.**Solution**Because c is negative, p and q must have different signs.

Factors of 7	Sum of factors	
$-7, 1$	$-7 + 1 = -6$	\times
$7, -1$	$7 + (-1) = 6$	\longleftarrow correct sum

The factors 7 and -1 have a sum of 6, so they are the correct values of p and q .

$$k^2 + 6k - 7 = (k + 7)(k - 1)$$

Exercises for Example 3

Factor the trinomial.

4. $x^2 - 10x - 11$

5. $y^2 + 2y - 63$

6. $z^2 - 5z - 36$

EXAMPLE 2 Factor when b is negative and c is positiveFactor $w^2 - 10w + 9$.**Solution**Because b is negative and c is positive, p and q must be negative.

Factors of 9	Sum of factors	
$-9, -1$	$-9 + (-1) = -10$	\longleftarrow correct sum
$-3, -3$	$-3 + (-3) = -6$	\times

The factors -9 and -1 have a sum of -10 , so they are the correct values of p and q .

$$w^2 - 10w + 9 = (w - 9)(w - 1)$$

Exercises for Examples 1 and 2

Factor the trinomial.

1. $x^2 + 10x + 16$

2. $y^2 + 6y + 5$

3. $z^2 - 7z + 12$

EXAMPLE 4 **Solve a polynomial equation**

Solve the equation $h^2 - 4h = 21$.

Solution

$$h^2 - 4h = 21$$

Write original equation.

$$h^2 - 4h - 21 = 0$$

Subtract 21 from each side.

$$(h + 3)(h - 7) = 0$$

Factor left side.

$$h + 3 = 0 \quad \text{or} \quad h - 7 = 0$$

Zero-product property

$$h = -3 \quad \text{or} \quad h = 7$$

Solve for h .

The roots of the equation are -3 and 7 .

Exercise for Example 4

7. Solve the equation $x^2 + 30 = 11x$.