

8.6 Factoring Trinomials

GOAL Factor trinomials of the form $ax^2 + bx + c$.

Key Concept

You can factor polynomials of the form $ax^2 + bx + c$ by finding the factors of a and the factors of c so that the sum of the outer and inner products is b .

Common Student Errors

- Giving up before trying all possible pairs of factors A visual like this may help:

Tip Remind students that factoring trinomials takes a lot of trial and error and with experience they will be able to quickly narrow down the possible factorizations.

$$ax^2 + bx + c = \left(\frac{?}{?}x + \frac{?}{?} \right) \left(\frac{?}{?}x + \frac{?}{?} \right)$$

factors of a (over the first $?$) and factors of c (under the last $?$)

Step 1 - Write the polynomial in standard form ($ax^2 + bx + c$)

Step 2 - Factor out the GCF (if possible)

Step 3 - () ()

Step 4 - (ax) (ax)

Step 5 - (ax \pm) (ax \pm)

To decide whether to use + or - , look at the **LAST + or - sign in your polynomial:

- a) If the **last** sign is -, the signs inside the parentheses will be **DIFFERENT**

$$(ax + \quad)(ax - \quad) \quad \text{or} \quad (ax - \quad)(ax + \quad)$$

- b) If the **last** sign is +, the signs inside the parentheses will be the **SAME** and you must look at the **first** sign in the polynomial to tell what they both will be:

•• If the first sign is + you will have (ax +)(ax +)

•• If the first sign is - you will have (ax -)(ax -)

Step 6 - $(ax \pm \square)(ax \pm \square)$

Factors of $\mathbf{a \cdot c}$ that add or subtract to equal \mathbf{b}

(If you made something illegal happen, you have to make it look like it never did! Better call in the boys, give him cement shoes, and make sure he's sleepin' with the fishes!)

- *Get rid of the GCF in each of the quantities...
do NOT write it outside the parentheses (that would leave evidence)
Just annihilate it - make it disappear forever!!
- **If you pulled a GCF out in front in step 2, LEAVE IT THERE,
do NOT annihilate it - you didn't do anything wrong by pulling it out to begin with!

Factor the Polynomial.

$$1. \frac{4x^2}{2} + \frac{10x}{2} + \frac{6}{2}$$

BOTH!

Step #5!

SAME

$$2(2x^2 + 5x + 3) *$$

$$\underline{A \cdot C = 2 \cdot 3 = 6}$$

$$2(\quad)(\quad)$$

$$\frac{+6}{2 \cdot 3} \text{ :D}$$

$$2(2x \quad)(2x \quad) \leftarrow \text{illegal}$$

$$2(2x + \quad)(2x + \quad)$$

$$2(\underline{2x+2})(2x+3)$$

$$\boxed{2(x+1)(2x+3)}$$

$$2. \quad \frac{6x^2}{3} + \frac{9x}{3} - \frac{6}{3}$$

diff.

*ignore negatives here.

$$3(2x^2 + 3x - 2) *$$

$$3(\quad)(\quad)$$

$$3(2x \quad)(2x \quad)$$

$$3(2x + \quad)(2x - \quad)$$

$$3\left(\frac{2x+4}{2}\right)(2x-1)$$

$$3(x+2)(2x-1)$$

$$\frac{a \cdot c = 4}{\begin{array}{c} 1 \cdot 4 \\ 2 \cdot 2 \end{array}}$$

EXAMPLE 1 Factor when b is negative and c is positive

Factor $5n^2 - 12n + 7$.

Solution

Because b is negative and c is positive, both factors of c must be negative. Make a table to organize your work.

You must consider the order of the factors of 7, because the x -terms of the possible factorization are different.

Factors of 5	Factors of 7	Possible factorization	Middle term when multiplied
1, 5	-1, -7	$(n - 1)(5n - 7)$	$-5n - 7n = -12n$
1, 5	-7, -1	$(n - 7)(5n - 1)$	$-n - 35n = -36n$

← correct

x

$$5n^2 - 12n + 7 = (n - 1)(5n - 7)$$

EXAMPLE 2 **Factor when b is negative and c is negative**

Factor $3m^2 - 5m - 22$.**Solution**Because b is negative and c is negative, p and q must have different signs.

Factors of 3	Factors of 22	Possible factorization	Middle term when multiplied	
1, 3	1, -22	$(m + 1)(3m - 22)$	$3m - 22m = -19m$	X
1, 3	-1, 22	$(m - 1)(3m + 22)$	$22m - 3m = 19m$	X
1, 3	2, -11	$(m + 2)(3m - 11)$	$-11m + 6m = -5m$	← correct
1, 3	-11, 2	$(m - 11)(3m + 2)$	$2m - 33m = -31m$	X

$$3m^2 - 5m - 22 = (m + 2)(3m - 11)$$

Exercises for Examples 1 and 2

Factor the trinomial.

1. $7a^2 - 50a + 7$

2. $4b^2 - 8b - 5$

3. $6c^2 + 5c - 14$

EXAMPLE 3 Factor when a is negative**Factor** $-2x^2 + 9x - 9$.**Solution****STEP 1** Factor -1 from each term of the trinomial.

$$-2x^2 + 9x - 9 = -(2x^2 - 9x + 9)$$

STEP 2 Factor the trinomial $2x^2 - 9x + 9$. Because b is negative and c is positive, both factors of c must be negative. Use a table to organize information about the factors of a and c .

Factors of 2	Factors of 9	Possible factorization	Middle term when multiplied	
1, 2	-1, -9	$(x - 1)(2x - 9)$	$-9x - 2x = -11x$	X
1, 2	-9, -1	$(x - 9)(2x - 1)$	$-x - 18x = -19x$	X
1, 2	-3, -3	$(x - 3)(2x - 3)$	$-3x - 6x = -9x$	← correct

$$-2x^2 + 9x - 9 = -(x - 3)(2x - 3)$$

Exercises for Example 3**Factor the trinomial.**

4. $-3r^2 - 7r - 4$

5. $-3s^2 + 8s + 16$

6. $-8t^2 + 6t - 1$