8.8 Factoring Polynomials

GOAL **Factor polynomials completely.**

Vocabulary

Factoring a common monomial from pairs of terms, then looking for a common binomial factor is called **factor by grouping.**

A polynomial of two or more terms is **prime** if it cannot be written as the product of polynomials of lesser degree using only integer coefficients and constants, and if the only common factors of its terms are 1 and –1.

A polynomial is factored completely if it is written as a monomial or as the product of a monomial (possibly 1 or -1) and one or more prime polynomials.

EXAMPLE 1 Factor out a common binomial

Factor the expression.

a. $5x^2(x-2)-3(x-2)$ **b.** $7y(5-y)+3(y-5)$

Solution

a. $5x^2(x-2)-3(x-2)=(x-2)(5x^2-3)$

b. The binomials $5-y$ and $y-5$ are opposites. Factor -1 from $5-y$ to obtain a common binomial factor.

$$7y(5-y)+3(y-5)=-7y(y-5)+3(y-5)$$ Factor -1 from $(5-y)$.

$$=(y-5)(-7y+3)$$ Distributive property

EXAMPLE 2 Factor by grouping

Factor the polynomial.

a. $m^3+7m^2-2m-14$ **b.** $n^3+30+6n^2+5n$

Solution

a. $m^3+7m^2-2m-14=(m^3+7m^2)+(-2m-14)$ Group terms.

$$=m^2(m+7)-2(m+7)$$ Factor each group.

$$=(m+7)(m^2-2)$$ Distributive property

b. $n^3+30+6n^2+5n=n^3+6n^2+5n+30$ Rearrange terms.

$$=(n^3+6n^2)+(5n+30)$$ Group terms.

$$=n^2(n+6)+5(n+6)$$ Factor each group.

$$=(n+6)(n^2+5)$$ Distributive property

Exercises for Examples 1 and 2

Factor the expression.

1. $11x(x-8)+3(x-8)$

2. $9x^3+9x^2-7x-7$

3. $10x^3+21y-35x^2-6xy$

EXAMPLE 4 Solve a polynomial equation

Solve the equation $7x^3 + 14x^2 = 105x$.

Solution

$7x^3 + 14x^2 = 105x$	Write original equation.
$7x^3 + 14x^2 - 105x = 0$	Subtract $105x$ from each side.
$7x(x^2 + 2x - 15) = 0$	Factor out $7x$.
$7x(x + 5)(x - 3) = 0$	Factor the trinomial.
$7x = 0$ *or* $x + 5 = 0$ *or* $x - 3 = 0$	Zero-product property
$x = 0$ *or* $x = -5$ *or* $x = 3$	Solve for x.

The roots of the equation are 0, -5, and 3.

Exercises for Example 3

Solve the equation.

4. $2c^3 + 8c^2 - 42c = 0$

5. $4x^3 + 48x^2 + 144x = 0$

6. $5r^3 + 15r = 20r^2$