9.1 Quadratic Equation

GOAL **Graph simple quadratic functions.**

Vocabulary

A **quadratic function** is a nonlinear function that can be written in the **standard form** $y = ax^2 + bx + c$ where $a \neq 0$.

Every quadratic function has a U-shaped graph called a **parabola.**

The most basic quadratic function in the family of quadratic functions, called the **parent quadratic function,** is $y = x^2$.

The lowest or highest point on a parabola is the **vertex.**

The line that passes through the vertex and divides the parabola into two symmetric parts is called the **axis of symmetry.**

Common Student Errors

- Confusing shrinks and stretches

  Tip Remind students that for the function $y = ax^2$, a determines how widely the parabola opens. When $|a|$ is small, the parabola opens more widely than when $|a|$ is large.

  Example: Compare the graphs of $y = 2x^2$ and $y = x^2$.

  Student response: The graph of $y = 2x^2$ is wider than the graph of $y = x^2$.

- Confusing vertical translations

  Tip Remind students that for the function $y = x^2 + c$, when $c > 0$, the graph shifts c units up and when $c < 0$, the graph shifts c units down.

  The following association may help:

  The *addition* of a positive number c to $y = x^2$ yields a *shift up* (in the *positive direction*). Whereas the *subtraction* of a positive number c yields a *shift down* (in the *negative direction*).

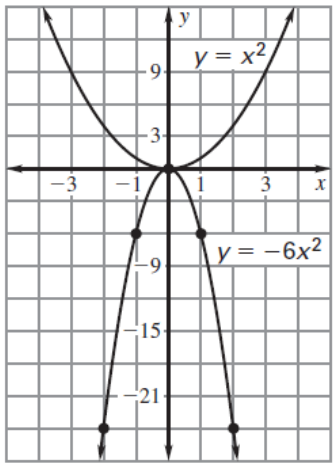
EXAMPLE 1 Graph $y = ax^2$ when $|a| > 1$

Graph $y = -6x^2$. Compare the graph with the graph of $y = x^2$.

Solution

STEP 1 **Make** a table of values for $y = -6x^2$.

x	-2	-1	0	1	2
y	-24	-6	0	-6	-24



STEP 2 **Plot** the points from the table.

STEP 3 **Draw** a smooth curve through the points.

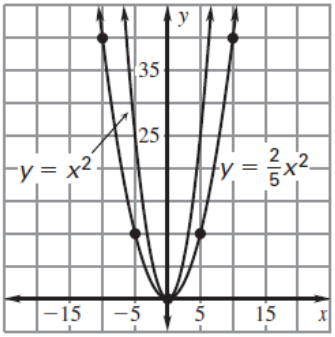
STEP 4 **Compare** the graphs of $y = -6x^2$ and $y = x^2$. Both graphs have the same vertex, $(0, 0)$, and the same axis of symmetry, $x = 0$. However, the graph of $y = -6x^2$ is narrower than the graph of $y = x^2$ and it opens down. This is because the graph of $y = -6x^2$ is a vertical stretch (by a factor of 6) of the graph of $y = x^2$ and a reflection in the x-axis of the graph of $y = x^2$.

EXAMPLE 2 Graph $y = ax^2$ when $|a| < 1$

Graph $y = \frac{2}{5}x^2$. Compare the graph with the graph of $y = x^2$.

STEP 1 **Make** a table of values for $y = \frac{2}{5}x^2$.

x	-10	-5	0	5	1	0
y	40	10	0	10	40	



STEP 2 **Plot** the points from the table.

STEP 3 **Draw** a smooth curve through the points.

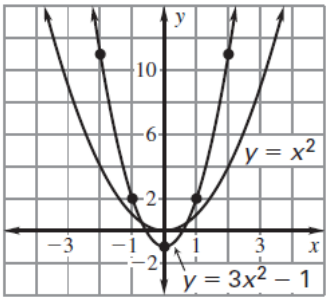
STEP 4 **Compare** the graphs of $y = \frac{2}{5}x^2$ and $y = x^2$. Both graphs have the same vertex, $(0, 0)$, and the same axis of symmetry, $x = 0$. Both graphs open upward. However, the graph of $y = \frac{2}{5}x^2$ is wider than the graph of $y = x^2$. This is because the graph of $y = \frac{2}{5}x^2$ is a vertical shrink $\left(\text{by a factor of } \frac{2}{5}\right)$ of the graph of $y = x^2$.

EXAMPLE 3 Graph $y = ax^2 + c$

Graph $y = 3x^2 - 1$. Compare the graph with the graph of $y = x^2$.

STEP 1 **Make** a table of values for $y = 3x^2 - 1$.

x	-2	-1	0	1	2
y	11	2	-1	2	11



STEP 2 **Plot** the points from the table.

STEP 3 **Draw** a smooth curve through the points.

STEP 4 **Compare** the graphs of $y = 3x^2 - 1$ and $y = x^2$. Both graphs open up and have the same axis of symmetry, $x = 0$. However, the graph of $y = 3x^2 - 1$ is narrower and has a lower vertex than the graph of $y = x^2$. This is because the graph of $y = 3x^2 - 1$ is a vertical stretch (by a factor of 3) and a vertical translation (1 unit down) of the graph of $y = x^2$.

Exercises for Examples 1, 2, and 3

Graph the function. Compare the graph with the graph of $y = x^2$.

1. $y = -8x^2$

2. $y = \frac{1}{7}x^2$

3. $y = -\frac{1}{3}x^2$

4. $y = x^2 - 3$

5. $y = \frac{1}{4}x^2 + 2$

6. $y = -\frac{1}{2}x^2 - 1$