11.2/11.3 Notes

GOAL

Use the formula for the number of permutations.

Vocabulary

A **permutation** is an arrangement of objects in which order is important.
For any positive integer *n*, the product of the integers from 1 to *n* is called ***n* factorial** and is written as *n*!.

The number of permutations of n objects taken r at a time is determined by the following formula:

$$P(n,r) = \frac{n!}{(n-r)!}$$

EXAMPLE 1

Count permutations

Consider the number of permutations of the letters in the word APRIL.

a. In how many ways can you arrange all of the letters?

b. In how many ways can you arrange 3 of the letters?

Solution

a. Use the counting principle to find the number of permutations of the letters in the word APRIL.

Number of Permutations		Choices for 1st letter		Choices for 2nd letter		Choices for 3rd letter		Choices for 4th letter		Choices for 5th letter
	=	5	•	4	•	3	•	2	•	1

= 120

There are 120 ways you can arrange all of the letters in the word APRIL.

b. When arranging 3 letters of the word APRIL, you have 5 choices for the first letter, 4 for the second letter, and 3 for the third letter.

Number of Permutations		Choices for 1st letter		Choices for 2nd letter		Choices for 3rd letter
	=	5	•	4	•	3

= 60

There are 60 ways you can arrange 3 of the letters in the word APRIL.

EXAMPLE 2

Use permutation formula

Packing You have 11 pairs of shorts and plan to pack 5 of them for a vacation. In how many ways can you choose the shorts you pack for your vacation?

Solution

To find the number of permutations of 5 pairs of shorts chosen from 11, find ${}_{11}P_5$.

${}_{11}P_5 = \dfrac{11!}{(11-5)!}$	Permutation formula
$= \dfrac{11!}{6!}$	Subtract.
$= \dfrac{11 \bullet 10 \bullet 9 \bullet 8 \bullet 7 \bullet \cancel{6!}}{\cancel{6!}}$	Expand factorials. Divide out common factorial, 6!.
$= 55{,}440$	Multiply.

There are 55,440 ways to arrange 5 pairs of shorts out of 11.

GOAL

Use combinations to count possibilities.

Vocabulary

A **combination** is a selection of objects in which order is *not* important.

In our example the order of the digits were important, if the order didn't matter we would have what is the definition of a combination. The number of combinations of n objects taken r at a time is determined by the following formula:

$$C(n,r) = \frac{n!}{(n-r)!r!}$$

EXAMPLE 1

Count combinations

Count the combinations of three letters from the list A, B, C, D.

Solution

List all of the permutations of three letters from the list A, B, C, D. Because order is not important in a combination, cross out any duplicate groupings.

ABC, ~~ACB~~, ABD, ~~ADB~~, ACD, ~~ADC~~
~~BAC~~, ~~BCA~~, BCD, ~~BDC~~, ~~BDA~~, ~~BAD~~
~~CAB~~, ~~CBA~~, ~~CBD~~, ~~CDB~~, ~~CAD~~, ~~CDA~~
~~DAB~~, ~~DBA~~, ~~DAC~~, ~~DCA~~, ~~DBC~~, ~~DCB~~

There are 4 possible combinations of 3 letters from the list A, B, C, D.